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The scattering of an acoustic line-source radiation from a rotating cylinder*

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The scattering of radiation of an acoustic linesource with harmonic time dependence, from a rotating cylinder has been investigated in detail, with special emphasis on the far scattered acoustic field. The analytical results obtained show the explicit dependence of far field on mode number n and the angular velocity of the rotating cylinder

1. INTRODUCTION

The problem of scattering of sound radiation from stationary/moving surfaces is an important problem in acoustics, due to its relevance to aviation acoustics and it can also be used in the identification of a scattering surface in case the scattered sound field is known The scattering of plane and spherical sound waves has been dealt with by many authors and an excellent review is given in the book of Morse & Ingrad (1968) Recently Samaddar (1973) has discussed the problem of scattering of acoustic line source radiation from a sphere It seems that the scattering of sound from a rotating cylinder has not been investigated so far. In general, a rotating cylinder drags the surrounding medium in contact with it due to viscosity effects and generates fluctuations in the static pressure distribution which in turn generate turbulent stresses in the surrounding medium As the inclusion of the above mentioned effects makes the problem almost intractable mathematically, one is forced to make use of certain approximations to get useful results

In this communication, we have analysed the scattering of radiation of an acoustic line source from a uniformly rotating cylinder, under the assumption that there is no interaction between the sound field generated by the line-source and the field generated in the medium by the rotating cylinder Expressions have been obtained for the far scattered sound field for both; a soft cylinder (the continuity of acoustic pressure at the scattering surface) and a hard cylinder

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(the continuity of radial velocity of acoustic particles at the scattering surface) cases The results obtained show clear dependence of the far scattered field on angular velocity of the rotating cylinder and mode number, n.

2 Formulation of the Problem

Let us assume an infinitely long cylinder of radius, a, placed at the origin of a cylindrical coordinate system (ρ, Φ, z) and is rotating with an angular velocity $\mathbf{\Omega} = \hat{z}\Omega$, $\hat{z}$ being a unit vector in z-direction; so that the scattering cylinder undergoes a uniform rotation with velocity $\boldsymbol{v}\phi = \mathbf{\Omega} \times \boldsymbol{\rho}$, $\boldsymbol{\rho}$ being the radius vector An infinitely long uniform acoustic line source with harmonic time dependence $\exp(-i\omega t)$, is assumed to drive the incident pressure field, p_i, which in turn satisfies the two-dimensional wave equation in cylindrical coordinates as

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\Phi^2}+k^2\right]p_i = \frac{Q}{c_0^2}\frac{\delta(\rho-\rho_0)}{\rho}\delta(\Phi) \quad . \quad (2.1)$$

where $k = \omega/c_0$, Q is the acoustic source strength, c_0 is the propagation velocity of the pressure waves in the medium, δ is Dirac delta function and the source is placed at $\rho = \rho_0$, $\Phi \equiv 0$ The harmonic times dependence $\exp(-\omega t)$ has been supressed in eq (2.1)

Assuming cylindrical wave like solutions of the form $\exp(i(n\Phi-\omega t)$, where, n, is the mode number, the solution of eq. (2.1) is

$$p_i(\rho,\Phi) = -\frac{Q}{c_0^2}\sum_{-\infty}^{\infty} J_n(k\rho_0)H_n^{(1)}(k\rho)\exp(i(n\Phi-\omega t)),\ \rho \geqslant \rho_0 \quad .. \quad (2.2a)$$

$$= -\frac{Q}{c_0^2}\sum H_n^{(1)}(k\rho_0)J_n(k\rho)\exp(i(n\Phi-\omega t)).\ \rho \leqslant \rho_0 \quad ... \quad (2.2b)$$

where J_n and $H_n^{(1)}$ are Bessel function and the Hankel function of first kind.

The scattered field, p_s, satisfies the general wave equation of moving media given by

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla\right)^2 p_s - c_0^2\nabla^2 p_s = 0 \quad ... \quad (2.3)$$

where $\boldsymbol{v} = \boldsymbol{v}\Phi$ and ∇ is the Laplacian operator in three-dimensions and p_s, satisfies the radiation condition at infinity along with boundary conditions at the scattering surface In our case the eq (2.3) reduces to

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)-\frac{n^2}{\rho^2}\right]p_s+\left(\frac{n\Omega}{c_0}-\frac{\omega}{c_0}\right)^2 p_s = 0 \quad ... \quad (2.4)$$

for cylindrical wave like solutions; the complete solution of the above equation for outgoing waves is

$$p_s(\rho, \Phi) = \sum_{-\infty}^{\infty} A_n H_n^{(1)}(K\rho)\exp(i(n\Phi - \omega t)) \qquad .. \quad (2.5)$$

with

$$K^2 = \left(\frac{n\Omega}{c_0} - k \right)^2.$$

The constants A_n are determined by the nature of the boundary condition at the scattering surface

3. Scattering from Hard and Soft Cylinders

The scattered sound field will be different through according as the scattering cylinder is soft or hard For the soft cylinder case, the total pressure of the incident and scattered sound field vanishes at the surface, i e ,

$$(p_i + p_s)_{\rho=a} = 0. \qquad \ldots \quad (3.1)$$

For a hard cylinder the total radial velocity of the particles of the medium vanishes at the scattering surface, i e ,

$$\left(\frac{\partial p_i}{\partial \rho} + \frac{\partial p_s}{\partial \rho} \right)_{\rho=a} = 0 \qquad \ldots \quad (3\ 2)$$

Soft cylinder case

Using eqs (2 2b), (2 5) and (3.1), the value of A_n is

$$A_n = \frac{Q}{c_0^2} \frac{H_n^{(1)}(k\rho_0)J_n(ka)}{H_n^{(1)}(Ka)},$$

so that the scattered sound field when expressed in spherical coordinates ($\rho = \gamma \cos \theta$) is

$$p_s(\gamma, \theta, \Phi) = \frac{Q}{c_0^2} \sum_{-\infty}^{\infty} \frac{H_n^{(1)}(k\rho_0)J_n(ka)}{H_n^{(1)}(Ka)} H_n^{(1)}(K\gamma \cos \theta)\exp(i(n\Phi - \omega t)) \qquad .. \quad (3\ 3)$$

If we consider the scattered sound field at a far point, when the argument of the Hankel function (3 3) yields

$$p_s(\gamma, \theta, \Phi) = \frac{Q}{c_0^2} \sum_{-\infty}^{\infty} \frac{H_n^{(1)}(k\rho_0)J_n(ka)}{H_n^{(1)}(Ka)} \exp(i(n\Phi - \omega t)$$

$$\times \sqrt{\frac{2}{\pi\gamma K \cos\theta}} \exp i\left(K\gamma \cos\theta - \frac{2n+1}{4} \pi \right) \qquad \ldots \quad (3.4)$$

Now for a finite value of cylinder radius and the finite location of the acoustic line-source ($k\rho_0$, ka, $Ka \ll 1$) after retaining first order smallness terms eq (3 4) yields

$$p_s(\gamma, \theta, \Phi) = \sqrt{\frac{i}{2\pi\gamma\cos\theta}}\, a^2 \exp(-i\omega t)$$

$$\times\left[\sqrt{\left(\frac{\Omega}{c_0}+k\right)}\exp i\left\{\left(\frac{\Omega}{c_0}+k\right)\gamma\cos\theta-\Phi\right\}\right.$$

$$-\sqrt{\left(\frac{\Omega}{c_0}-k\right)}\exp i\left\{\left(\frac{\Omega}{c_0}-k\right)\gamma\cos\theta+\Phi\right\}$$

$$\left.-\frac{2}{a^2}\frac{\ln(k\rho_0)}{\ln(ka)}\exp(ik\gamma\cos\theta)\right] \qquad \ldots \quad (3.5)$$

Hard cylinder case

In case the scattering cylinder is hard, using eqs (2 2b), (2 5) and (3 2), the value of A_n comes out as

$$A_n = \frac{Q}{c_0^2}\frac{H_n^{(1)}(k\rho_0)J'_n(ka)}{H_n^{(1)\prime}(Ka)},$$

where prime denotes differentiation with respect to argument Hence the scattered field, p_s, in spherical coordinates ($\rho = \gamma\cos\theta$) is

$$p_s(\gamma, \theta, \Phi) = \frac{Q}{c_0^2}\sum_{-\infty}^{\infty}\frac{H_n^{(1)}(k\rho_0)J'_n(ka)}{H_n^{(1)\prime}(Ka)}H_n^{(1)}(K\gamma\cos\theta)\exp(i(n\Phi-\omega t)) \qquad \ldots \quad (3\ 6)$$

For a finite value of cylinder radius and with the finite location of the acoustic line-source the above equation becomes

$$p_s(\gamma, \theta, \Phi) = \sqrt{\frac{2i}{\pi\gamma\cos\theta}}\;\frac{a^2\exp(-i\omega t)}{k\rho_0}\times\left[\left(\frac{\Omega}{c_0}-k\right)^{3/2}\exp i\left\{\left(\frac{\Omega}{c_0}-k\right)\right.\right.$$

$$\left.\gamma\cos\theta+\Phi\right\}+\frac{\sqrt{k}}{2\rho_0}\exp(ik\gamma\cos\theta)-\left(\frac{\Omega}{c_0}+k\right)^{3/2}\exp i\left.\left\{\left(\frac{\Omega}{c_0}+k\right)\gamma\cos\theta-\quad\right\}\right]$$

$$\ldots \quad (3.7)$$

4 Discussion

Eqs (3 4) through (3 7) are the desired expressions for the far scattered acoustic field, under a highly idealized condition that line source field and the field generated due to the rotation of the cylinder do not interact It may be noted that the results obtained are valid for non-zero values of K^2 (i.e $n\Omega/c_0 \neq k$) and can be easily computed if so desired The results also show the explicit dependence of far scattered acoustic field, on the mole number, n, and the angular velocity modulus Ω, of the rotating cylinder

References

Morse P. M. & Ingrad K. U. 1968 *Theoretical Acoustics*, McGraw-Hill, New York
Sammadar J. N. 1973 *J. Sound and Vibration* **27**, 271.